MATH 551 - Problem Set 2

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March 20, 2022

4.2. Could not figure out.

4.3

(a) By Ceva's theorem, we know that if the product of the ratio of divided sides in a triangle is 1, then the lines that intersect those sides must be concurrent. Well, we are given all of these ratios and it is true that $\frac{k3}{k2} \times \frac{k2}{k1} \times \frac{k1}{k3} = 1$ simply from simplification, and so by Ceva's theorem we know that AP_{BC}, BP_{CA}, and CP_{AB} are concurrent.

(b) By Menelaus' theorem, we know that if the product of the ratio of divided sides in a triangle is -1, then the lines that intersect those lines (that is, the sides of the triangle extended) must be concurrent. We know that the ratio of the two divided sides with respect to the internal points are $\frac{k2}{k1}$ and $\frac{k1}{k3}$ and we know that the ratio of the third is the negative of the internal ratio because it's outside of the triangle, so $-\frac{k3}{k2}$, and because $-\frac{k3}{k2} \times \frac{k2}{k1} \times \frac{k1}{k3} = -1$, we know (by Menelaus) that the points P'_{BC}, P_{CA}, and P_{AB} are concurrent.

(c) Again, we use Menelaus. This time, knowing that the ratio of all of the segments involved is the negation of their internal ratio, we find that $-\frac{k3}{k2} \times -\frac{k2}{k1} \times -\frac{k1}{k3} = -1$, which means we may conclude, by Menelaus, that P'_{BC}, P'_{CA}, and P'_{AB} are concurrent.

4.4

Because we know that $AM = PC$ and $PC = LB$, we know that $AM = BL$ which means that we may write $M - A = L - B$ which means that $\frac{M+B}{2} = \frac{L+A}{2}$, therefore BM and AL bisect each other. The same is true (by symmetry, for two other cases), that is $CL = AN$ or $L - C = N - A$ so $\frac{L+A}{2} = \frac{N+C}{2}$ which means that AL and CN bisect each other. And finally $MC = NB$ or $C - M = B - N$ so $\frac{C+N}{2} = \frac{B+M}{2}$ which means that CN and BM bisect each other. Now we see (by the transitive property), that $\frac{L+A}{2} = \frac{N+C}{2} = \frac{B+M}{2}$, which means that AL, BM, and CN all bisect each other.

5.3 Could not figure out.